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Abstract. Higher-order thinking skill is conceived as the top end of Bloom's cognitive taxonomy. Cognitive domain of higher order thinking skill consists of analyzing, evaluating, and creating. The study aimed to describe the type of error made by vocational school students in solving problems involving higher order thinking skills based on Newman, Lai, Subanji & Mulyoto. This study employed a qualitative descriptive approach with a test as data collecting technique. The respondents are 12 students of vocational school. The skill domain of analyzing contributes reading & decoding error, comprehension error, conceptual error, process skills error, and encoding error. The highest percentage of students' error in this domain is conceptual error (33%). The skill domain of evaluating contributes decoding & reading error, comprehension error, language interpretation error, process skills error, and encoding error. The highest percentage of students' error in this domain is encoding error (25%). The skill domain of creating contributes conceptual error, comprehension error, and process skill error. The highest percentage of students' error in this domain is conceptual error (42%). The results suggest mathematics teacher to explore deeply about their students' skill of analyzing, skill of evaluating, and skill of creating.

1. Introduction

Since 2006, Indonesian Government had set out the standard of content in regulation number 22 of 2006. The regulation state that the need of mathematics to be taught in school is to equip students with the ability to think logically, analytically, systematically, critically, and creatively [1]. Besides that, 21st century skills require students to be able to think critically, creatively, and be able to solve problems [2]. Critical and creative thinking skills are some basic skills that included in higher order thinking skills [3].

Higher order thinking skill occur when individual getting a new information, store, arranging, as well as finding the relation between the existing knowledge and extending the information to fulfill the objectives and solving complex situation [4]. Higher order thinking skill demand someone to apply new information or knowledge that he/she has got and manipulates the information to reach possibility of answer in a new situation [5]. The characteristics of higher order thinking skill are related to critical and creative thinking skills [1]. Higher order thinking skill is a major component of creative and critical thinking and creative thinking pedagogy that can help students to develop more innovative ideas, ideal perspectives, and imaginative insights [6]. Table 1 shows Fisher's characterization of higher order thinking skills versus routine teaching [7]. It can be concluded that higher order thinking skill is a major

component of creative and critical thinking that requires a person to apply the new information or knowledge to achieve possible answers in a new situation.

Table 1. Characteristics of higher order thinking skill versus routine teaching

Higher Order Thinking Skill	Routine Teaching
Non Routine	Routine
Involving Uncertainty	Seeking Certainty
Complex	Clear Goal and Purpose
Producing Multiple Solutions / Open Ended	Producing Converging Outcomes
Making Meaning Process	Process of Doing
Effortful (Mental Work Required)	Judged by Outcome (Rather Than Effort)

In 1956, Bloom introduced conceptual framework of higher order thinking skill named bloom's taxonomy [8]. Bloom's taxonomy is hierarchical structure that identify thinking skills from the lowest to the highest level. In 1994, student of Bloom named Krathwohl, and cognitive psychologist named Anderson, repaired Bloom's taxonomy. This change was happened to cognitive domain. This result was published in 2001 by Anderson & Ktrathwohl with the comparison between old bloom's taxonomy and bloom's taxonomy revision is as follows [9].

Table 2. The comparison between old bloom's taxonomy and bloom's taxonomy revision

Old Bloom's Taxonomy	Revision of Bloom's Taxonomy
C1 (Knowledge)	C1 (Remember)
C2 (Comprehension)	C2 (Understand)
C3 (Application)	C3 (Apply)
C4 (Analysis)	C4 (Analyse)
C5 (Synthesis)	C5 (Evaluate)
C6 (Evaluation)	C6 (Create)

Each level in bloom's taxonomy revision is changing. Each level changes from noun to verb. Moreover, the steps are also changed, such as evaluation that before standing on C6 in old bloom's taxonomy, become C5 in the bloom's taxonomy revision. While the position of C5 (synthesis) rises to C6 in bloom's taxonomy revision and there is a fundamental change of the term from synthesis become creating.

Cognitive domain of higher order thinking skill consists of analyze, evaluate, and create [10]. The process are in the level of C4, C5, and C6 of bloom's taxonomy revised by Anderson & Krathwohl [9]. Some experts also agree that higher-order thinking is conceived as the top end of bloom's cognitive taxonomy [5]. While in the knowledge domain, higher order thinking skill consists of conceptual knowledge, procedural knowledge, and metacognitive knowledge [9].

2. Methods
The method of this research is descriptive-qualitative. Qualitative research often involves a rich collection of data from various sources to gain a deeper understanding of individual participants [11]. This research emphasizes on analyzing students' mistakes in solving mathematics problems involving higher order thinking skill.

2.1. Error analyzing in solving mathematics problem
Cheng-Fei Lai identifies the reason why students may create error [12]. The first reason is lack of knowledge. Lack of knowledge contributes three types of error: procedural, factual, and conceptual error. Procedural error happen when a student has not followed the correct steps or procedures to solve a problem. Factual error happen when students cannot recall a fact required to solve a problem. Conceptual errors may look like procedural errors, but they occur because the student does not fully

understand a specific math concept. The second reason is poor attention and carelessness**.** Teachers should first consider the alignment between the instruction, student ability, and the task to address this problem.

Newman has introduced a simple model to determine the students' problems in solving the mathematics questions which is called Newman's Error Analysis (NEA) [13]. There are five stages in recognizing students' error based on NEA model. The errors are i) Reading and Decoding, ii) Comprehension, iii) Transformation, iv) Process Skills and v) Encoding.

Table 3. Newman Error Analysis (NEA) Stages

Error domain	Description
Reading and Decoding	The students' ability in reading the problem given and determining the words or symbols given in questions
Comprehension	The students' understanding related to the symbols, expressions, and problems given in the questions
Transformation	The ability of students in choosing the appropriate formulae or method to solve the problems given
Process Skill	Exploring the process skills of the students in solving the problems whether the method or operation they use are correct or wrong
Encoding	The ability of the students in generating and justifying the answer they give

Subanji & Mulyoto state the type of errors in solving mathematics problems [14]. They introduce five basic errors: conceptual error, data using error, language interpretation error, technical error, and conclusion generating error.

1. Conceptual Error
   It consists of (a) error in determining theorems or formulas in problem solving and (b) the use of theorem or formula is not in accordance with the conditions of the prerequisite.
2. Data Using Error
   It consists of (a) error in using unrelated data, (b) error entering data into variables, and (c) error in adding data that is not needed to solve the problem.
3. Language Interpretation Error
   It consists of (a) Error in reflecting daily language to mathematics language and (b) error in interpreting symbols, graphs, and table to mathematics language.
4. Technical Error
   It consists of (a) calculation error and (b) error in manipulating algebraic operations.
5. Conclusion Generating Error
   It consists of (a) generating conclusion without the correct supporting and (b) generating conclusion that is not in accordance with logical reasoning.

Table 4. Error analysis in HOTS domain

Newman (1977)	Lai (2012)	Subanji & Mulyoto (Romadiastri, 2017)	Error analysis in HOTS domain		
			Analysing	**Evaluating**	**Creating**
	Factual Error				

Reading and Decoding Error		Conceptual Error	Reading & Decoding Error	Decoding & Reading Error	Conceptual Error
Comprehension Error		Data Using Error	Comprehension Error	Comprehension Error	Comprehension Error
Transformation Error	Conceptual Error	Language Interpretation Error	Conceptual Error	Language Interpretation Error	X
Process Skills Error	Procedural Error	Technical Error	Process Skills Error	Process Skills Error	Process Skills Error
Encoding Error		Conclusion Generating Error	Encoding Error	Encoding Error	X

2.2. Constructing higher order thinking skills test instrument

There are three task forms in measuring higher order thinking skills: (1) Selection, which includes multiple choice, matching, rank-order item, (2) Generalization, which includes essay and short-answer. (3) Explanation, which includes reason for choosing [15]. Nitko and Brookhart state that multiple choice will eliminate the opportunity to express students' ideas [16]. Besides that, multiple choice tests can be insignificant and limited to factual knowledge. By answering essay test, students not only recognize information, but recall it. Essay test involved recalling process, but multiple choice only used recognition process [17]. Thus, this research used essay test to examine students' higher order thinking skills.

Based on bloom's taxonomy revision, there are three basic indicators of higher order thinking skills: analyze, evaluate, and create. Each Indicator of higher order thinking skills consists of some sub-indicators.

Table 5. Indicator and sub-indicator of higher-order thinking skills [1]

Indicator of HOTS	Sub-indicator of HOTS	Cognitive Domain
Analyze (C4)	Distinguish	
	Organize	
	Attribute	
Evaluate (C5)	Check	Conceptual
	Criticize	Procedural
Create (C6)	Formulate	Metacognitive
	Plan	
	Produce	

Table 6. Sub-indicator and questions of higher-order thinking skills in exponent and root topic

Sub-indicator	Question
Organize procedure of simplifying roots, procedure of solving roots, and procedure of rationalizing denominator to solve the problem	Solve $\dfrac{\sqrt{3}}{\sqrt{20}-\sqrt{5}+\sqrt{32}-3\sqrt{2}}$ and rationalize the denominator if needed.
Organize procedure of exponent multiplication and procedure of logarithm to solve the problem	Solve $^{2}\log\left[\dfrac{8^{2}\times 2^{3}}{16}\right]$
Criticize procedures of solving problem related to exponent	In a mathematics competition, the committee give final question to the three finalists. The finalists are asked to simplify $\dfrac{12a^{6}b^{2}}{2a^{2}b^{6}}$. The answers of those three finalists are as follows.

a. Finalist 1

$$\frac{12a^6b^2}{2a^2b^6} = \frac{12}{2} a^{\frac{6}{2}}b^{\frac{2}{6}} = 6a^3b^{\frac{1}{3}}$$

b. Finalist 2

$$\frac{12a^6b^2}{2a^2b^6} = \frac{12}{2} a^{6-2}b^{2-6} = 6a^4b^{-4}$$

c. Finalist 3

$$\frac{12a^6b^2}{2a^2b^6} = (12-2)a^{6-2}b^{2-6} = 10a^4b^{-4}$$

Circle the wrong steps of the finalists' answers, then give the reasons. Who deserves to be the champion of the competition?

Produce mathematics statement involve exponent, multiplication, and division	Suppose you are a math teacher. Make questions that at least involve exponent, multiplication, and division operation having the following results.
	a. 4
	b. x^3
	c. a^2b

Validity and reliability are important aspects in constructing instrument of the test. A test has validity if it measures what it purports to measure. Type of validity of this instrument is content validity. Content validity is established through a rational analysis of the test content, and its determination is based on individual or subjective judgment [18]. There are two mathematics teacher evaluating this instrument. The result indicates that this test instrument is valid.

A test is reliable if its observed scores are highly correlated with its true score [18]. The reliability of this instrument is determined by internal-consistency reliability with coefficient of α (cronbach). If the scores of the halves have unequal variances or there is some other indication that the halves not parallel, coefficient of α (cronbach) can be used to estimate the reliability of the whole test [18]. This instrument coefficient of α (cronbach) is 0.65. The reliability score indicates that this instrument is reliable.

After the instrument of the test was declared valid and reliable, it was tested to students. The test were applied to 12 vocational school students. They were all 11th graders majoring in nursing. They had already learn exponent, root, and logarithm material in 10th grade.

3. Result and discussion

After solving the questions, students collect their worksheet to the researcher. After checking all students' answers, the result shows that each student make mistake or error. The students' error is varies greatly.

Table 7. Percentage of students' error in each domain

Analyzing		Evaluating		Creating	
Type of Error	Percentage	Type of Error	Percentage	Type of Error	Percentage
Reading & Decoding Error	8%	Decoding & Reading Error	17%	Conceptual Error	42%
Comprehension Error	17%	Comprehension Error	17%	Comprehension Error	17%
Conceptual Error	33%	Language Interpretation Error	17%	Process Skills Error	17%

| Process Skills Error	17%	Process Skills Error	8%
		Encoding Error	25%

3.1. Categorization of error based on skill of analyzing

Figure 1. Reading, decoding, and process skills error

Figure 2. Comprehension error

Figure 3. Conceptual error

Figure 4. Encoding error

Figure 1 indicates that the student is not careful in reading the question. They write $\sqrt{5}$ instead of $\sqrt{3}$. Based on NEA, the error is categorized as reading and decoding error [13]. The student also makes a simple mistake in calculating $5 - 2 = -3$. Based on NEA, it is categorized as process skills error [13]. Figure 2 indicates that student does not understand the question holistically. The question ask student to rationalize the denominator if needed, but the student does not solve that. Based on NEA, the error belong to comprehension error [13]. Figure 3 indicates that the student has no concept understanding of $\sqrt{a} - \sqrt{b}$. The error occur when the student calculate $\sqrt{20} - \sqrt{5} = \sqrt{15}$. Moreover, student shows incorrect concept in simplifying $\sqrt{15}$ become $3\sqrt{5}$. Based on Lai, Subanji and Mulyoto, the error belong to conceptual error [12,14]. Figure 4 shows that the student solve the problem using correct procedure, but at last the student write incorrect statement of $^2\log 32 = {}^2\log 5$. The statement should be $^2\log 32 = 5$. Based on NEA, the error belong to encoding error [13]. The highest percentage of students' error in this domain is conceptual error (33%).

3.2. Categorization of error based on skill of evaluating

Figure 5. Reading, decoding, and language interpretation error

Figure 6. Comprehension error and encoding error
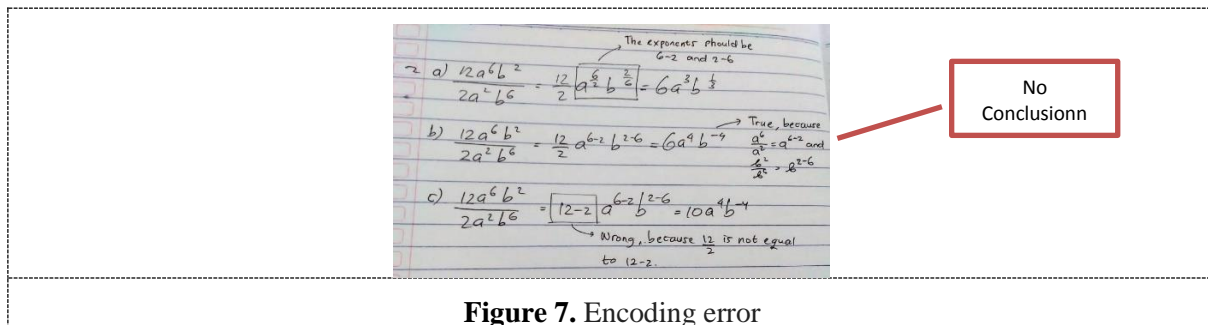
Figure 7. Encoding error

Figure 5 indicates that the student is not able to understand the question. The question ask students to circle the wrong steps of the finalist answers, but the students make a circle in the letter. Based on NEA, the error is categorized as reading and decoding error [13]. Besides that, the error is also categorized as language interpretation error based on Subanji and Mulyoto [14]. Figure 6 indicates that the student gives correct answer, but no reason and conclusion. The student does not state the champion of the competition. Based on NEA, the error is categorized as comprehension and encoding error [13]. Figure 7 shows that the student gives the correct answer and reason but no conclusion. The student does not state the champion of the competition. Based on NEA, the error is categorized as encoding error [13]. The highest percentage of students' error in this domain is encoding error (25%).

3.3 Categorization of error based on skill of creating

Figure 8. Comprehension error

Figure 9. Conceptual error

Figure 10. Conceptual error

Figure 11. Process skills error

Figure 8 indicates that the student is able to create some questions making correct result, but the student do not understand the question holistically. The question ask students to use at least involve exponent, multiplication, and division operation. Based on NEA, the error is categorized as comprehension error [13]. Figure 9 indicates that the student does not understand the concept of addition and exponent. The student makes error when making equation of $a^2 = a + a$. Figure 10 showing $x + x^2 = x^3$ also indicates that the student doesn't understand the concept of addition and exponent. Based on Cheng, Subanji, and Mulyoto, the error can be categorized as conceptual error. Figure 11 indicates that the student gives incorrect statement of $a^2b = \dfrac{a^5b^2}{a^3b^2}$. Incorrect calculation also happen when the student writes $b = \dfrac{b^2}{b^2}$.

According to NEA, the error is categorized as process skills error. The highest percentage of students' error in this domain is conceptual error (42%).

4. Conclusions

Cognitive domain of higher order thinking skill consists of analyze, evaluate, and create. There are some types of errors when students solve the problem involving analyzing, evaluating, and creating. The error happen in all domain. The domain of the skill of analyzing contributes reading and decoding error,

comprehension error, conceptual error, process skills error, and encoding error. The highest percentage of students' error in this domain is conceptual error (33%). The domain of the skill of evaluating contributes decoding & reading error, comprehension error, language interpretation error, and encoding error. The highest percentage of students' error in this domain is encoding error (25%). The domain of the skill of creating contributes conceptual error, comprehension error, and process skill error. The highest percentage of students' error in this domain is conceptual error (42%). The results suggest mathematics teacher to explore deeply about their students' skill of analyzing, skill of evaluating, and skill of creating, that all includes in higher order thinking skill.

5. References

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